

**NUMERICAL PRESSURE CALCULATION OF UNSTEADY-STATE RADIAL FLOW
ON SLIGHTLY COMPRESSIBILITY FLUIDS
(PERHITUNGAN TEKANAN SECARA NUMERIK UNTUK ALIRAN RADIAL TIDAK
STABIL PADA FLUIDA SEDIKIT KOMPRESIBILITAS)**

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ABSTRACT

The numerical pressure calculation has been calculated for the radially unstable fluid flow in a slightly compressible fluid, using fortran soft program and the results are interpreted in the form of a graph of the function of drainage radius and time. This paper also explains how to derive the pressure equation mathematically which fulfills the mass conservation principle, so the fluid continuity equation is obtained using radial / spherical coordinates and the diffusivity equation. By using the solution of the Ei function, the fluid flow in the radial direction of the porous media can be shown. This mathematical derivation is considered important because in unsteady state fluids, the time variable is very important to consider. It can be seen in the resulting graph, the pressure at the outer point has a maximum value then it will decrease as the drainage radius to the well decreases and with increasing time according to the nature of unsteady state flows.

Keyword: *Pressure, Numerical calculation, Unsteady-state flow, Radial flow, Slightly compressibility fluids.*

ABSTRAK

Sudah dihitung perhitungan tekanan secara numerik pada aliran fluida radial tidak stabil pada fluida sedikit kompresibilitas, dengan menggunakan soft program fortran dan hasilnya diinterpretasikan dalam bentuk grafik fungsi jari-jari drainase dan waktu. Pada paper ini juga dijelaskan cara menurunkan persamaan tekanan secara matematis dimana prinsip konservasi massa dipenuhi, maka didapatkan persamaan kontinuitas fluida dengan menggunakan koordinat radial / bola dan persamaan difusi. Dan dengan menggunakan solusi fungsi Ei digambarkan aliran fluida arah radial pada media berpori. Penurunan matematika disini dianggap penting karena pada unsteady state fluids variable waktu menjadi sangat penting untuk dipertimbangkan. Terlihat pada grafik yang dihasilkan, pada titik terluar tekanannya maksimum kemudian akan menurun seiring berkurangnya jari-jari drainase dan bertambahnya waktu sampai ke sumur. sesuai dengan sifat aliran yang tidak stabil.

Kata Kunci: *Tekanan, Perhitungan numerik, Aliran radial, Aliran tidak stabil, Fluida sedikit kompresibilitas.*

I. INTRODUCTION

In research on fluid flow in an oil and gas reservoir, it is found that most of the flow is unsteady-state, for some cases it is simplified to steady state, such as the decrease in pressure equations for linear and radial flows published (Listiana, 2018.1). This research is a continuation of the calculation of a stable flow rate into an unstable flow calculation. The obvious difference between these two types of flow is that the flow rate is not stable depending on the time variable. The unstable flow equation is derived from the steady flow equation with some additions or changes. In a stable flow, the amount of fluid entering the system equals the amount of fluid that leaves. In an unstable flow, the flow rate into a volume element of a porous medium may not be the same as the outflow rate of the element, because the amount of flow that flows will change with time, this does not happen in steady-state flows. In addition, the porosity ϕ and compressibility C will affect this unstable flow. The compressibility properties of fluid consist of 3, namely compressibility, slightly compressibility and incompressibility. For incompressibility is an ideal condition so that there may not be any fluid that meets, and for fluid compressibility is usually found in gas, for water and crude oil it is usually slightly compressibility. Therefore, here will be discussed about the slightly compressibility fluid. And there are 2 types of oil flow in the reservoir, namely linear and radial flow, this research is devoted to radial flow because most of the flow in the reservoir is radial flow.

II. THEORY

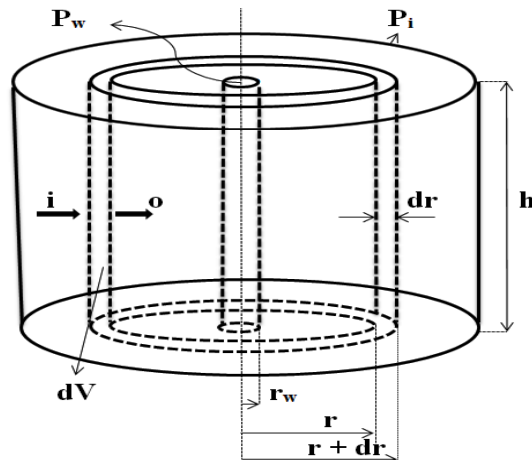


Figure 1. Volume elements in the well

Figure 1 is a picture of a well where dV is the volume element (ft^3) which will be used to calculate the material balance concept, the distance r (ft) from the center of the well and the width is dr . P is pressure (psi) where the index w is the pressure on the well and i is the external pressure, and r and h are the radius and height of the reservoir. Whereas i and o are fluid flows that enter and leave the volume elements. According to the concept of material balance, the mass flow rate into the element (m_{in}) minus the mass outflow rate in a certain time interval (m_{out}) is equal to the accumulated mass rate at that time interval.

$m_{in} - m_{out}$ = the rate of mass accumulation over the time interval τ (1)

The mass that enters the volume element during the time interval τ is

$$m_{in} = \Delta\tau(A v \rho)_{r+dr} \quad (2)$$

where, mass (lb), v fluid flow velocity (ft / days), ρ is the fluid density at $r + dr$ (lb / ft³), A is the area at $r + dr$ (ft²) and Δt is the time interval in days respectively.

The entry area on the element is the area of the cylindrical blanket

$$A = 2\pi h.(r + dr) \quad (3)$$

Equation (3) is inserted into Equation (2)

$$m_{in} = 2\pi h.\Delta\tau.(r + dr).(v \rho)_{r+dr} \quad (4)$$

In the same way the mass flow rate leaving the volume element is obtained during the time interval τ is

$$m_{out} = 2\pi h.\Delta\tau.r.(v\rho)_r \quad (5)$$

Volume with radius r is

$$V_r = \pi r^2.h$$

Whereas the volume element is the derivative of the volume with respect to distance / radius

$$dV_r = 2\pi h.r dr \quad (6)$$

The total mass accumulated during $\Delta \tau$ is

$$dV_r . [(\emptyset \rho)_{\tau+dr} - (\emptyset \rho)_{\tau}] = 2\pi r.h dr . [(\emptyset \rho)_{\tau+dr} - (\emptyset \rho)_{\tau}] \quad (7)$$

Where $\emptyset$ is porosity (fraction) and ρ is density (lb/ft³)

Equation (1) becomes

$$2\pi h.\Delta\tau (r + dr) [v \rho]_{r+dr} - 2\pi h.\Delta\tau r[v \rho]_r = 2\pi h.rdr[(\emptyset \rho)_{\tau+dr} - (\emptyset \rho)_{\tau}]$$

Divided by $2\pi h.rdr$ to become

$$\frac{1}{rdr} [(r + dr)[v \rho]_{r+dr} - r[v \rho]_r] = \frac{1}{\Delta\tau} [(\phi \rho)_{\tau+d\tau} - (\phi \rho)_{\tau}]$$

$$\frac{\partial}{r\partial r} [r v \rho] = \frac{\partial}{\partial \tau} (\phi \rho) \quad (8)$$

Equation (8) is called the Equation of Continuity, proves the principle of Conservation of Mass using radial / spherical coordinates
The Darcy equation in radial / spherical form is:

$$v = 5.615 \times 0.00112 \frac{k \partial p}{\mu \partial r} = 0.006328 \frac{k \partial p}{\mu \partial r} \quad (9)$$

with μ is viscosity (cp) and k is permeability (md)

Substitute Equation (9) to Equation (8),

$$0.006328 \frac{\partial}{r\partial r} \left[\frac{\rho r k}{\mu} \frac{\partial p}{\partial r} \right] = \frac{\partial(\phi \rho)}{\partial \tau} \quad (10)$$

The right part is described,

$$\frac{\partial(\phi \rho)}{\partial \tau} = \frac{\phi \partial \rho}{\partial \tau} + \frac{\rho \partial \phi}{\partial \tau} \quad (11)$$

The relationship between porosity and compressibility of the formation is

$$c_f = \frac{\partial \phi}{\phi \partial p} \quad (12)$$

$$\frac{\partial \phi}{\partial \tau} = \frac{\partial \phi}{\partial p} \cdot \frac{\partial p}{\partial \tau}$$

By using the chain rule of differentiation, and:

$$\frac{\partial \phi}{\partial \tau} = \frac{\phi c_f \partial p}{\partial \tau}$$

Equation (11) becomes :

$$\frac{\partial(\phi \rho)}{\partial \tau} = \frac{\phi \partial \rho}{\partial \tau} + \frac{\rho \phi c_f \partial p}{\partial \tau}$$

dan equation (10) becomes ;

$$0.006328 \frac{\partial}{r\partial r} \left[\frac{\rho r k}{\mu} \frac{\partial p}{\partial r} \right] = \frac{\phi \partial \rho}{\partial \tau} + \frac{\rho \phi c_f \partial p}{\partial \tau} \quad (13)$$

The so-called General Differential Equation is used to describe fluid flow in the radial direction of a porous medium.

To simplify Equation (13), it is assumed that the permeability and viscosity are constant for a certain range of pressure, time, and distance. So that

$$0.006328 \frac{k}{r\mu} \frac{\partial}{\partial r} \left[r \rho \frac{\partial p}{\partial r} \right] = \frac{\phi \partial \rho}{\partial \tau} + \frac{\rho \phi c_f \partial p}{\partial \tau} \quad (14)$$

$$0.006328 \frac{k}{\mu} \frac{\partial}{\partial r} \left[r \rho \frac{\partial p}{\partial r} \right] = \frac{\phi \partial \rho}{\partial \tau} + \frac{\rho \phi c_f \partial p}{\partial \tau}$$

$$\frac{0.006328k}{\mu} \left[\frac{\rho \partial p}{r \partial r} + \frac{\rho \partial^2 p}{\partial r^2} + \frac{\partial p}{\partial r} \frac{\partial \rho}{\partial r} \right] = \frac{\phi \partial \rho}{\partial \tau} + \frac{\rho \phi c_f \partial p}{\partial \tau}$$

By using the chain rule

$$\frac{0.006328k}{\mu} \left[\frac{\rho \partial p}{r \partial r} + \frac{\rho \partial^2 p}{\partial r^2} + \left(\frac{\partial p}{\partial r} \right)^2 \frac{\partial \rho}{\partial p} \right] = \frac{\rho \phi c_f \partial p}{\partial \tau} + \phi \cdot \left(\frac{\partial p}{\partial \tau} \right) \left(\frac{\partial \rho}{\partial p} \right)$$

Then divided by the density of the liquid ρ

$$\frac{0.006328k}{\mu} \left[\frac{\partial p}{r \partial r} + \frac{\partial^2 p}{\partial r^2} + \left(\frac{\partial p}{\partial r} \right)^2 \left(\frac{\partial \rho}{\rho \partial p} \right) \right] = \frac{\phi c_f \partial p}{\partial \tau} + \frac{\phi}{\rho} \left(\frac{\partial p}{\partial \tau} \right) \left(\frac{\partial \rho}{\partial p} \right)$$

The relationship between compressibility and fluid density is

$$c = \frac{\partial \rho}{\rho \partial p}$$

The equation above becomes

$$\frac{0.006328k}{\mu} \left[\frac{\partial p}{r \partial r} + \frac{\partial^2 p}{\partial r^2} + \left(\frac{\partial p}{\partial r} \right)^2 c \right] = \frac{\phi c_f \partial p}{\partial \tau} + \phi c \left(\frac{\partial p}{\partial \tau} \right)$$

The form $\left(\frac{\partial p}{\partial r} \right)^2$ is considered too small to be ignored

$$\frac{0.006328k}{\mu} \left[\frac{\partial p}{r \partial r} + \frac{\partial^2 p}{\partial r^2} \right] = \phi (c_f + c) \frac{\partial p}{\partial \tau} \quad (15)$$

The total compressibility of c_t is defined as,

$$c_t = c + c_f \quad (16)$$

From Equation (14), (15), and (16),

$$\frac{\partial^2 p}{\partial r^2} + \frac{\partial p}{r \partial r} = \frac{\phi \mu c_t \partial p}{0.006328k \partial \tau} \quad (17)$$

Equation (17) is called the Diffusivity equation, this equation is used in the analysis of well testing data. τ is calculated in days, if in hours, then Equation (17) becomes,

$$\frac{\partial^2 p}{\partial r^2} + \frac{\partial p}{r \partial r} = \frac{\phi \mu c_t \partial p}{0.0002637 k \partial \tau} \quad (18)$$

Where, k is permeability (md), r is radial position (ft), p is pressure (psia), c_t is total compressibility (psi^{-1}), t is time (days or hours), ϕ is porosity (fraction) and μ is viscosity (cp). The diffusion equation above states pressure as a function of time τ and position r .

Matthews and Russel (1967) propose a solution to the Diffusivity Equation mentioned above, using the Ei Function Solution:

$$p(r, \tau) = p_i + \left[\frac{70.6 \times Q_o \mu B_o}{k h} \right] Ei \left[\frac{-948 \times \phi \mu c_t r^2}{k \tau} \right] \quad (19)$$

Where, $p(r, \tau)$ is the pressure on the radius r from the well after time t (hours), k is permeability (md) and Q_o is the flow rate (STB / day).

Ei's math function is

$$Ei(-x) = A_1 + A_2 \ln x + A_3 [\ln x]^2 + A_4 [\ln x]^3 + A_5 x + A_6 x^2 + A_7 x^3 + \frac{A_8}{x} \quad (20)$$

The index x is

$$x = \frac{948 \times \phi \mu c_t r^2}{k t} \quad (21)$$

The values of the coefficient are:

$$\begin{aligned} A_1 &= -0.33153973 \\ A_2 &= -0.81512322 \\ A_3 &= 0.0522123384 \\ A_4 &= 0.0059849819 \\ A_5 &= 0.662318450 \\ A_6 &= -0.12333524 \\ A_7 &= 0.010832566 \\ A_8 &= 0.00086709776 \end{aligned}$$

III. RESULTS AND DISCUSSION

If an oil well produces at a constant flow rate (Q_o) of 300 STB / day under unsteady state and the reservoir has rock and fluid properties (Tarek Ahmed, 2012) & (Tarek Ahmed, 2001), with the code for the computer program in brackets [] :

$$B_o [B0] = 1,25 \text{ bbl/STB}$$

$$k_o [k] = 60 \text{ md}$$

$$\phi [por] = 15 \%$$

$$\mu_o [\mu] = 1,5 \text{ cp}$$

$$h [h] = 15 \text{ ft}$$

$$r_w [r] = 0,25 \text{ ft}$$

$$c_t [ct] = 12 \times 10^{-6} \text{ psi}^{-1}$$

$$p_i [pi] = 4000 \text{ psi}$$

Then the pressure (psi) as a function of position (ft) and time (hr) is

$$p(r, t) = p_i + \left[\frac{70,6Q_0\mu B_0}{kh} \right] Ei \left[\frac{-948\phi\mu c_t r^2}{kt} \right]$$

Fortran computer program (Tanja van Mourik, 2005)& (Listiana, 2018.2):
for 1, 12, and 24 hour time periods

$$a1 = -0.33153973$$

$$a2 = -0.81512322$$

$$a3 = 5.22123384d-2$$

$$a4 = 5.9849819d-3$$

$$a5 = 0.662318450$$

$$a6 = -0.12333524$$

$$a7 = 1.0832566d-2$$

$$a8 = 8.6709776d-4$$

$$\text{do } r = 1, 1401, 1$$

$$x = (948.0Do*por*\mu*ct*r*r)/(k*t)$$

$$Ei = -(a1+a2*\log(x)+a3*(\log(x))^{**2}+a4*(\log(x))^{**3}+a5*x+a6*x^{**2}+a7*x^{**3}+a8/x)$$

$$prt = pi+((70.6Do*Qo*\mu*Bo)/(k*h))*Ei$$

$$\text{if}(t.\text{eq}.1.\text{and}.r.\text{gt}.286) \text{ prt} = pi$$

$$\text{if}(t.\text{eq}.12.\text{and}.r.\text{gt}.976) \text{ prt} = pi$$

$$\text{if}(t.\text{eq}.24.\text{and}.r.\text{gt}.1381) \text{ prt} = pi$$

end do

the resulting graph is

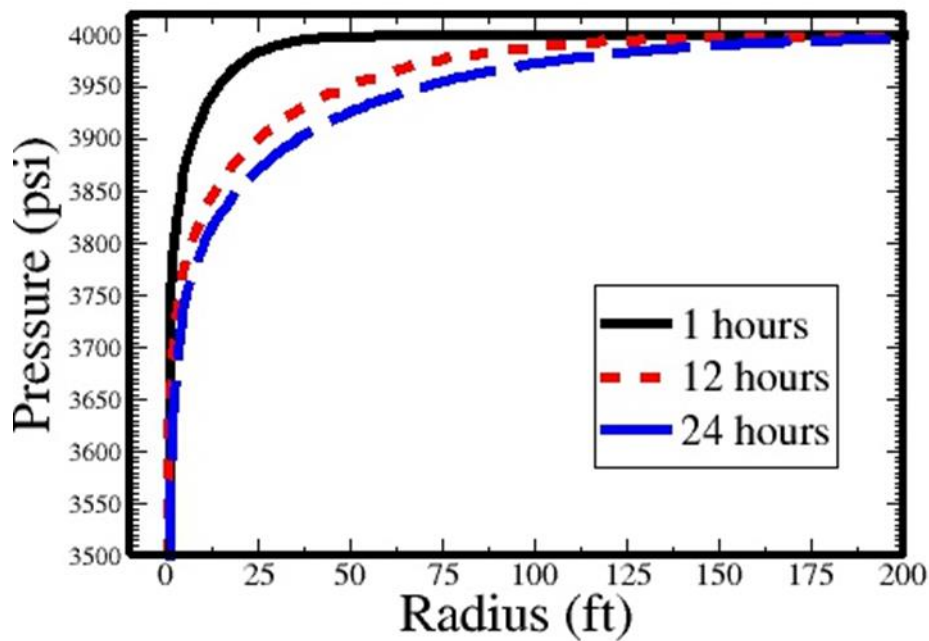


Figure 2. The relation between pressure and fingers at various intervals of time

Figure 2 is the numerical calculation result of the pressure on the reservoir using formula (19), in the graph it can be seen that the smaller the radius or the closer to the well, the lower the pressure, this occurs in steady state and unsteady state flows. But if the time variable is entered, it will be seen from the graph that the pressure is not only affected by radius but also affected by time. This can be seen on the graph, at a longer time, starting from 1 hour, 12 hours, and 24 hours, the pressure will decrease. This is what happens in conditions in the field for a longer time, so the pressure will decrease. For the steady state flow that has been discussed in the past by Listiana (2018.1), constant pressure is not affected by time, this is an ideal condition not a real condition.

IV. CONCLUSIONS

1. The calculation of the pressure resulting from the reduction in the formula from the principle of mass conservation on the flow of unsteady state and the fluid with slightly compressibility is obtained.
2. Calculated by numerical calculations to get results in accordance with the unsteady state flow properties, namely the pressure will change with changes in the drainage radius and time, this is shown in the resulting graph. This differs from steady state flow where the pressure is not affected by time.
3. Besides depending on the drainage radius and time, according to the formula solution obtained, the pressure is also influenced by the compressibility and porosity factors of the reservoir.

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